METRIZABILITY AND SUBMETRIZABILITY FOR POINT-OPEN, OPEN-POINT AND BI-POINT-OPEN TOPOLOGIES ON C(X, Y

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Abstract

The article about the metrizability and submetrizability for point-open, open-point and bi-point-open topologies on c(x, y) oftheform[x,V]−={f∈C(X,Y):f(x)∈V},whereV isanopensetinY and x ∈ X. The space C(X, Y) equipped with point-open topology is denoted by Cp(X,Y).

Key words: isanopensetin, point topology, subbase consisting

The open-point topology h [3] on C(X,Y) has a subbase consisting of the setsoftheform[U,y]−={f∈C(X,Y):f−1(y)∩U =φ},whereU isanopen set in X and y ∈ Y . The space C(X, Y) equipped with open-point topology is denoted by Ch(X, Y).

The bi-point-open topology ph [3] has subbasic open sets of both kinds: [U,y]− = {f ∈ C(X,Y) : f−1(y)∩U = φ} and [x,V]+ = {f ∈ C(X,Y) : f(x)∈V},whereUisanopensetinXandy∈Y;x∈XandV isopen in Y . The space C(X, Y) equipped with bi-point-open topology is denoted by Cph(X,Y).

A nonempty subset of a space X is said to be Gδ-dense [3] provided that it intersects every nonempty Gδ-subset of X.

Throughout this paper, X is a completely regular and Hausdorff space and (Y,+,×) denotes a locally convex Hausdorff space, where + and × denote vector addition and scalar multiplication, respectively. r × y is a scalar mul- tiplication of r and y, where r ∈ R and y ∈ Y. Let −y denote additive inverseofpointyinY and−U={−1×u:u∈U},where−u=−1×u. V + U = {v + u : u ∈ U, v ∈ V }, where u + v denotes the vector addition in Y . Let 0 denote the identity element in the space Y . Let N and R denote the set of all natural numbers and set of all real numbers, respectively. P denotes the collection of all seminorms that generates the topology of Y . Let X and Y denote two spaces with same underlying set, and X ≤ Y denote that X and Y have the same topology. Also, 0X denotes the constant function which maps all points of X to 0. |Z| denotes the cardinality of set Z. Let X0 denote the set of all isolated points in X.

x1,U1]+ ∩ ··· ∩ [xn,Un]+. For each B ∈ B, let K(B) = {x1,...,xn}. Let T = SB∈B K(B). Clearly, |T | ≤ χ(Cp(X, Y), 0X). Let if possible, there exists x0 ∈ X \T. Then let [x0,V1(p)] be a subbasic open set in Cp(X,Y), where p ∈ P. Consider an arbitrary B ∈ B, where B = [x1,U1]+ ∩ ··· ∩ [xn,Un]+. Since X is completely regular, there exists a continuous function f : X → R suchthatf(x0)=1andf(xi)=0foralli∈1,...,n. Takey∈Y,where p(y) = 1. Also, since Y is a locally convex space, there exists a continuous functionh:R→Y definedbyh(a)=ay. Sothecompositionh◦f:X→Y is also continuous, where h ◦ f(xi) = 0 for all i ∈ 1,...,n and h ◦ f(x0) = y. Note that, h ◦ f ∈ B \ [x0 , V1 (p)]. This contradicts the fact that B forms a base at 0X forCp(X,Y).

Corollary 3.2. If Cp(X, Y) is first countable, then X is countable. Theorem 3.3. If Ch(X, Y) is first countable, then X has a countable π-base.

Proof. Since Ch(X, Y) is first countable, Ch(X, Y) has a countable pseudochar- acter. Then there exists countable family α of open sets in Ch(X, Y) such that { 0X } = ∩α. Without loss of generality, we can assume that each member A of α is of the form [V1,x1]∩···∩[Vn,xn], where for each i ∈ {1,...,n}, Vi is open inXandxi ∈Y. LetK(A)={V1,...,Vn}andletB=SA∈αK(A). Note that, B is a countable family of open sets. We show that B forms a π-base for the space X. Let V be an open set in X containing x. Then there exists a continuous function f : X → R such thatf(x) = 1 and f(v) = 0 for all v in Vc. Takey∈Y,wherep(y)=1forsomep∈P. SinceY isalocallyconvex space, there exists a continuous function h : R → Y defined by h(a) = ay, where p(y) = 1. So the composition h ◦ f : X → Y is also continuous, where h◦f(v)= 0forallvinVc andh◦f(x)=y. ThereexistsA∈αsuchthat h ◦ f ∈/ A. It follows that there exists U ∈ B such that 0 ∈/ g ◦ f (U). We claim that, U ⊂ V . Let u ∈ U, then g ◦ f(u) = 0, this implies u ∈ V . Hence U ⊂ V and B forms a π-base for X.

Recall that, for any space X and any point $x \in X$, the pseudocharac- ter of x in X, denoted by $\psi(X,x)$, is defined by $\psi(X,x) = \aleph 0 + \min\{|\gamma| : \gamma$ is a family of nonempty open subsets in X such that $\bigcap \gamma = \{x\}\}$.

The pseudocharacter $\psi(X)$ of X is given by

$\psi(X) = \sup\{\psi(X,x) : x \in X\}$.

If Cp(X, Y) is metrizable, then Y is metrizable. But a locally convex space Y is metrizable).

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