# GAS DYNAMICS EQUATIONS REPRESENTED BY THE BURGERS EQUATION

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**Abstract.** This thesis briefly describes the modeling of gas dynamics through the Burgers equation, which represents non-linear wave processes. Issues of mathematical modeling of wave propagation in gas dynamics are mentioned.

**Key words:** dissipative effect, Burgers equation, entropy, viscosity, gas dynamics.

Byurgers tenglamasini aniq fizik masala uchun qaraylik. Gaz dinamikasi tenglamalariga murojaat qilaylik. Qaralayotgan boshlang'ich tenglamalar sistemasi qovushqoqlik

$$\rho(v_t + vv_x) = -p_x + \eta v_{xx}, \qquad (1)$$

va uzluksizlik

$$\rho_t + (\rho v)_x = 0 \qquad (2)$$

hisobga olingan holda harakat tenglamasidan iborat bo'ladi. Bu yerda $\rho$ - gaz zichligi, $v$ - tezlik, $p$ - bosim, $\eta$ - qovushqoqlik koeffitsienti. Bosim quyidagi holat tenglamasi bo'yicha zichlikka bog'liq

$$p = k\rho^\gamma. \qquad (3)$$

Albatta, bu holda oqimni izentropik deb hisoblash mumkin emas. Biroq, akustik to'lqinda entropiya o'zgarishlari kichik bo'lib, bizni qiziqtiradigan yaqinlashish doirasida (3) tenglamadan foydalanishimiz mumkin. Bosimni buzilmagan holatga yaqin $\rho = \rho_0$ kvadratik hadlargacha Teylor qatoriga yoyamiz

$$p = p_0 + c_s^2(\rho - \rho_0) + \frac{c_s^2(\gamma-1)}{2\rho_0}(\rho - \rho_0)^2 + \cdots, \qquad (4)$$

bu yerda $c_s = \sqrt{\frac{\partial p}{\partial \rho}}$ - chiziqli tovush to'lqinlarining tezligi.

Birinchi navbatda chiziqli to'lqinlarning tarqalishini qaraylik. Kichik tebranishlar $\tilde{v}, \tilde{\rho}$ nisbatan (1), (2) tenglamalarni chiziqlashtirsak va $\tilde{v}, \tilde{\rho} \sim e^{i(\omega t - kx)}$ deb faraz qilsak,

$$\omega^2 = c_s^2 k^2 + \frac{i\eta}{\rho_0}\omega k^2 \qquad (5)$$

dispersiya munosabatiga kelinadi. Kuchsiz dissipatsiya holatini qaraylik, agar (5) tenglamaning o'ng tomonidagi ikkinchi had kichik bo'lsa, o'ng tomonga tarqaladigan to'lqinlar uchun

$$\omega \approx c_s k + \frac{i\nu k^2}{2} \qquad (6)$$

munosabatga ega bo'lamiz, bu yerda $v = \eta/\rho_0$ - kinematik qovushqoqlik.

Bu holda $\xi, \tau$ o'zgaruvchilar qanday kiritilishi kerakligini bilish uchun quyidagi faza hisoblanadi

$$\theta = \omega t - kx = k(c_s t - x) + \frac{ivk^2 t}{2}. \qquad (7)$$

Quyidagi o'lchamsiz o'zgaruvchilarni kiritamiz

$$v' = \frac{v}{c_s}, \quad \rho' = \frac{\rho}{\rho_0}, \quad t' = \frac{c_s^2 t}{v}, \quad x' = \frac{c_s x}{v}.$$

U holda (4) ni hisobga olgan holda (1) va (2) tenglamalar

$$\rho(v_t + vv_x) = -\left[\rho + (\gamma - 1)\frac{(\rho-1)^2}{2}\right]_x + v_{xx} \qquad (8)$$

$$\rho_t + (\rho v)_x = 0 \qquad (9)$$

va (7) munosabat ko'rinishini quyidagicha yozish mumkin:

$$\theta = k(t - x) + ik^2 t/2.$$

$k = \varepsilon^p \kappa$ belgilash kiritib, $\theta = \varepsilon^p \kappa(t - x) + i\varepsilon^{2p}\kappa^2 t/2$ tenglikka ega bo'lamiz. Shunday qilib, quyidagi shaklda yangi o'zgaruvchilar tanlanishi kerak

$$\xi = \varepsilon^p(x - t), \ \tau = \varepsilon^{2p} t \,.$$

$$v = \sum_{i=1}^{\infty} \varepsilon^i v_i \,, \ \ \rho = 1 + \sum_{i=1}^{\infty} \varepsilon^i \rho_i,$$

Yig'indilarni (8), (9) tenglamalarga qo'yib, $\xi$, $\tau$ o'zgaruvchilar bo'yicha quyidagi ifodaga ega bo'lamiz

$$\varepsilon^{p+1}(-v_{1\xi} + \rho_{1\xi}) +$$
$$+\varepsilon^{p+2}(-\rho_1 v_{1\xi} + v_1 v_{1\xi} - v_{2\xi} + \rho_{2\xi} + (\gamma - 1)\rho_1 \rho_{1\xi}) + \cdots$$
$$\ldots + \varepsilon^{2p+1}(v_{1\tau} - v_{1\xi\xi}) + \cdots = 0,$$

$$\varepsilon^{p+1}(-\rho_{1\xi} + v_{1\xi}) + \cdots + \varepsilon^{p+2}(-\rho_{2\xi} + v_{2\xi} + (\rho_1 v_1)_\xi) + \cdots$$
$$\ldots + \varepsilon^{2p+1}\rho_{1\tau} + \cdots = 0.$$

Cheksizlikda nol chegaraviy shartlar bilan integrallashdan keyin $\varepsilon^{p+1}$ tartib shartlari quyidagi ifodani beradi

$$\rho_1 = v_1 \qquad (10)$$

Trivial bo'lmagan yechimni olish uchun keyingi tartibda *p*=1 ni tanlash kerak, shunda 2*p*+1=*p*+2. Shunday qilib, $\varepsilon^3$ tartib shartlari (10) ni hisobga olgan holda

$$v_{1\tau} - v_{1\xi\xi} + (\gamma - 1)v_1 v_{1\xi} - v_{2\xi} + \rho_{2\xi} = 0,$$
$$v_{1\tau} + 2v_1 v_{1\xi} + v_{2\xi} - \rho_{2\xi} = 0$$

tenglamalarga kelinadi. Ushbu tenglamalarni qo'shish orqali Burgers tenglamasi olinadi

$$v_{1\tau} + \left(\frac{\gamma+1}{2}\right) v_1 v_{1\xi} = \frac{1}{2} v_{1\xi\xi} \qquad (11)$$

E'tibor berish kerakki, dispersiya munosabati (6) o'rniga bir xil darajadagi aniqlikni yozish mumkin

$$k \approx \frac{\omega}{c_s} - \frac{i\nu\omega^2}{2c_s^3}.$$

U holda $\theta = \omega(t - x/c_s) - i\nu\omega^2 x/2c_s^3$. O'lchamsiz koordinata va vaqtga o'tilsa

$$\tau = \frac{\varepsilon c_s(c_s t - x)}{\nu}, \quad \xi = \frac{\varepsilon^2 c_s x}{\nu},$$

Byurgers tenglamasining boshqa formasiga kelish mumkin

$$v_{1\xi} - \left(\frac{\gamma+1}{2}\right) v_1 v_{1\tau} = \frac{1}{2} v_{1\tau\tau}$$

Bu shakldа tenglama chegaraviy shart (signalning tarqalishi masalasi) bilan masalani tahlil qilish uchun qulayroqdir.

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